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A NEUMANN PROBLEM FOR AN ELASTIC MATERIAL UNDER ANTIPLANE STRESS

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ABSTRACT

An elastic semi-infinite layer occupying the region $0 \leq x < \infty, 0 \leq y \leq a$ under out-of-plane stress is investigated. The problem is formulated using conformal mapping and Mellin transform and solved by method of residue. A closed form solution for the displacement is obtained as a solution of the Neumann problem. The shear stress states at the interior of the material are determined from the displacements. The stress component at the point $y = \frac{a}{2}$ in the interior of the layer is dependent on the applied load T while the displacement vanishes at the same point indicating that cracking will start at the interior of the layer where $y = \frac{a}{2}$, as the applied load T becomes very large.

Keywords: Mellin transform, conformal mapping, Neumann boundary value problem, semi-infinite layer, anti-plane stress.

1. INTRODUCTION

The Neumann problem for an elastic cylinder under out-of-plane loading was investigated by Nnadi [1]. Using complex variable methods, he obtained the desired stress intensity factor. Sneddon [2] analyzed the distribution of potential in an elastic wedge without a crack. The stress intensity factor for a wedge under out-of-plane stress was calculated by Emenogu and Nnadi [3], Ouchterlony [4] carried out the analysis of an infinite wedge subjected to concentrated loads. Employing the Wiener Hoph technique, the asymptotic expressions for the stresses are derived.

Theocaris and Dafemos [5] analyzed a rectangular strip under plane stress. Using Mellin transform, Theocaris and Makrakis [6] solve the problem of crack kinking in anti-plane shear.

In this paper we consider a semi-infinite elastic, isotropic layer of height a units subjected to self-equilibrating anti plane shear loads of magnitude T on the length of the strip. The geometry of the problem is shown in fig. 1. In respect to the Cartesian coordinate system, the equation of motion is the Laplace's equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \tag{1}$$

The loading induces the boundary conditions

$$\sigma_{xz}(x, y) = 0 \tag{2}$$

$$\sigma_{yz}(x, y) = \pm T, \quad 0 \leq x \leq \infty, \quad 0 \leq y \leq a \tag{3}$$

The problem is to investigate the stress state at the interior of the layer to determine possible crack initiation point.

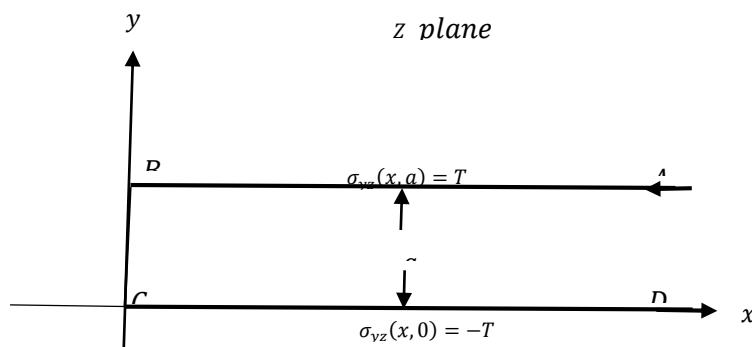


Fig. 1. The geometry of the problem

2. SOLUTION OF THE PROBLEM

To make the problem analyzable by method of integral transformation, the original z -plane of analysis is transformed onto the w -plane by the holomorphic function

$$w(z) = \cosh\left(\frac{\pi}{a}\right)z, \quad z = x + iy, \quad w = u + iv \quad (4)$$

Applying the holomorphic function, we have

$$\begin{aligned} w(z) &= \cosh\frac{\pi}{a}(x + iy) = \cosh\frac{\pi}{a}x \cosh\frac{\pi}{a}iy - \sinh\frac{\pi}{a}x \sinh\frac{\pi}{a}iy \\ &= \cosh\frac{\pi}{a}x \cos\frac{\pi}{a}y - i \sinh\frac{\pi}{a}x \sin\frac{\pi}{a}y \\ \therefore u(x, y) &= \cosh\frac{\pi}{a}x \cos\frac{\pi}{a}y \quad \text{and} \quad v(x, y) = -\sinh\frac{\pi}{a}x \sin\frac{\pi}{a}y \end{aligned} \quad (5)$$

Using eqn.(5), we obtain the transformed plane as

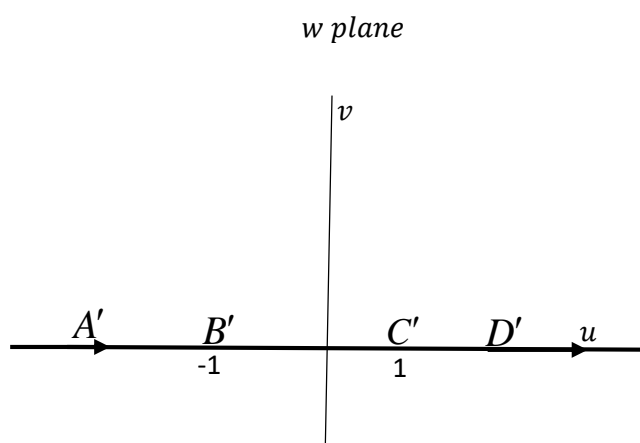


Fig.2. Showing the transformed w -plane in the upper half plane

From the stress-displacement relation $\sigma_{xz}(x, y) = \mu \frac{\partial w(x, y)}{\partial x}$ and $\sigma_{yz}(x, y) = \mu \frac{\partial w(x, y)}{\partial y}$

(6)

we have

$$\frac{\partial w(x, y)}{\partial x} = 0, \quad \frac{\partial w(x, y)}{\partial y} = \pm \frac{T}{\mu} \quad (7)$$

From the conformality condition $\phi(x, y) = w(u, v)$, the governing equation together with the boundary condition becomes

$$\frac{\partial^2 w}{\partial u^2} + \frac{\partial^2 w}{\partial v^2} = 0 \quad (8)$$

$$\frac{\partial w(u, 0)}{\partial v} = -\frac{aT}{u\pi} \left(e^{\frac{\pi x}{a}} \right)^{-1} \quad (9)$$

$$\frac{\partial w(u, a)}{\partial v} = \frac{aT}{u\pi} \left(e^{\frac{\pi x}{a}} \right)^{-1} \quad (10)$$

In polar form, eqns.(8),(9) and (10) becomes

$$\frac{\partial^2 w(r, \theta)}{\partial r^2} + \frac{1}{r} \frac{\partial w(r, \theta)}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w(r, \theta)}{\partial \theta^2} = 0, \quad r = e^{\frac{\pi x}{a}}, \quad \theta = \frac{\pi}{a} y \quad (11)$$

$$\frac{\partial w(r,0)}{\partial \theta} = -\frac{aT}{\mu\pi}$$

(13)

Next, we apply the Mellin transform [7] defined by

$$\bar{w}(s, \theta) = \int_0^{\infty} w(r, \theta) r^{s-1} dr \quad (14)$$

to eqns. (11),(12) and (13) to get

$$\frac{d^2 \bar{w}(s, \theta)}{d\theta^2} + s^2 \bar{w}(s, \theta) = 0 \quad , \quad -1 < \text{Re } s < 1 \quad (15) \quad \frac{\partial \bar{w}(s,0)}{\partial \theta} = -\frac{aT}{\mu\pi s}$$

(16)

$$\frac{\partial \bar{w}(s, \pi)}{\partial \theta} = -\frac{aT}{\mu\pi s} \quad (17)$$

Solving eqn.(15), we consider a solution of the form

$$\bar{w}(s, \theta) = p(s) \cos s\theta + Q \sin s\theta \quad (18)$$

To obtain the values of the unknown coefficients $p(s)$ and $Q(s)$, we have

$$\frac{\partial \bar{w}(s, \theta)}{\partial \theta} = -sp \sin s\theta + sQ \cos s\theta \quad (19)$$

$$\frac{\partial \bar{w}(s, 0)}{\partial \theta} = -sp \sin s(0) + sQ \cos s(0) = -\frac{aT}{\mu\pi s}$$

$$= sQ = -\frac{aT}{\mu\pi s}$$

$$\therefore Q = -\frac{aT}{\mu\pi s^2} \quad (20)$$

Also

$$\frac{\partial \bar{w}(s, \pi)}{\partial \theta} = -sp \sin s(\pi) + sQ \cos s(\pi) = -\frac{aT}{\mu\pi s}$$

$$= -sp \sin s(\pi) - s \left(\frac{aT}{\mu\pi s^2} \right) \cos s(\pi) = -\frac{aT}{\mu\pi s}$$

$$sp \sin s(\pi) = \frac{aT}{\mu\pi s} - \left(\frac{aT}{\mu\pi s} \right) \cos s(\pi)$$

$$sp \sin s(\pi) = \frac{aT}{\mu\pi s} [1 - \cos s\pi]$$

$$p = \frac{aT}{\mu\pi s^2} \left[\frac{1 - \cos s\pi}{\sin s\pi} \right] \quad (21)$$

Next, we substitute eqns.(20) and (21) into eqn.(18) to obtain

$$\begin{aligned} \bar{w}(s, \theta) &= \frac{aT}{\mu\pi s^2} \left[\frac{1 - \cos \pi s}{\sin \pi s} \right] \cos \theta s - \frac{aT}{\mu\pi s^2} \sin s\theta \\ &= \frac{aT}{\mu\pi s^2} \left[\left(\frac{1 - \cos \pi s}{\sin \pi s} \right) \cos \theta s - \sin s\theta \right] \end{aligned}$$

$$\begin{aligned}
&= \frac{aT}{u\pi s^2} \left[\frac{\cos \theta s - \cos \pi s \cos \theta s - \sin \pi s \sin \theta s}{\sin \pi s} \right] \\
&= \frac{aT}{u\pi s^2} \left[\frac{\cos \theta s - [\cos \pi s \cos \theta s + \sin \pi s \sin \theta s]}{\sin \pi s} \right] \\
&= \frac{aT}{u\pi s^2} \left[\frac{\cos \theta s - \cos(\pi - \theta)s}{\sin \pi s} \right] \tag{22}
\end{aligned}$$

Once eqn.(14) is satisfied, the displacement sought for is given by the inverse of the Mellin transform given by

$$w(r, \theta) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} r^{-s} \bar{w}(s, \theta) ds, \quad -1 < c < 1 \tag{23}$$

Hence, we have

$$w(r, \theta) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} r^{-s} \frac{aT}{\mu\pi s^2} \left[\frac{\cos \theta s - \cos(\pi - \theta)s}{\sin \pi s} \right] ds, \quad -1 < c < 1$$

Application of residue theory and Jordan's lemma, leads to

$$w(r, \theta) = \frac{aT}{\mu\pi} \left[\frac{\pi}{2} - \theta + \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} (\cos n\theta - \cos(\pi - \theta)^n) \right] r^n, \quad r < 1 \tag{24}$$

$$= \frac{aT}{\mu\pi} \left[\frac{1}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} (\cos n\theta - \cos(\pi - \theta)^n) \right] r^{-n}, \quad r > 1 \tag{25}$$

3.0 Analysis of the stress field

For $r < 1$

$$w(r, \theta) = \frac{aT}{\mu\pi} \left[\frac{\pi}{2} - \theta + \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} (\cos n\theta - \cos n(\pi - \theta)) \right] r^n, \quad 0 \leq \theta \leq \pi$$

$$\frac{\partial w(r, \theta)}{\partial \theta} = \frac{aT}{\mu\pi} \left[-1 + \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} (-\sin n\theta - \sin n(\pi - \theta)) \right] r^n, \quad 0 \leq \theta \leq \pi$$

$$\therefore \sigma_{\theta z}(r, \theta) = \frac{\mu}{r} \frac{\partial w}{\partial \theta} = \frac{aT}{\pi} \left[-1 - \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} (\sin n\theta + \sin n(\pi - \theta)) \right] r^{n-1} \tag{26}$$

For $r > 1$

$$w(r, \theta) = \frac{aT}{\mu\pi} \left[\frac{1}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} (\cos n\theta - \cos n(\pi - \theta)) \right] r^{-n}$$

$$\frac{\partial w(r, \theta)}{\partial \theta} = -\frac{aT}{\mu\pi^2} \left[\sum_{n=1}^{\infty} \frac{(-1)^n}{n} (\sin n\theta + \sin n(\pi - \theta)) \right] r^{-n}$$

$$\sigma_{\theta z} = \frac{\mu}{r} \frac{\partial w(r, \theta)}{\partial \theta} = -\frac{aT}{\pi^2} \left[\sum_{n=1}^{\infty} \frac{(-1)^n}{n} (\sin n\theta + \sin n(\pi - \theta)) \right] r^{-n-1}$$

For $r < 1$

$$\begin{aligned}
w(r, \theta) &= \frac{aT}{\mu\pi} \left[\frac{\pi}{2} - \theta - \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} (\cos n\theta - \cos n(\pi - \theta)) \right] r^n, \quad 0 \leq \theta \leq \pi \\
\frac{\partial w(r, \theta)}{\partial r} &= \frac{aTn}{\mu\pi} \left[\frac{\pi}{2} - \theta - \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} (\cos n\theta - \cos n(\pi - \theta)) \right] r^{n-1} + r^n (0) \\
\sigma_{rz}(r, \theta) &= \mu \frac{\partial w}{\partial r} = \frac{naT}{\pi} \left[\frac{\pi}{2} - \theta + \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} (\cos n\theta - \cos n(\pi - \theta)) \right] r^{n-1} \quad (27)
\end{aligned}$$

For $r > 1$

$$\begin{aligned}
w(r, \theta) &= \frac{aT}{\mu\pi} \left[\frac{1}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} (\cos n\theta - \cos n(\pi - \theta)) \right] r^{-n} \\
\frac{\partial w(r, \theta)}{\partial r} &= \frac{-naT}{\mu\pi} \left[\frac{1}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} (\cos n\theta - \cos n(\pi - \theta)) \right] r^{-n-1} \\
\sigma_{rz}(r, \theta) &= \mu \frac{\partial w}{\partial r} = \frac{-aT}{\pi^2} \left[\sum_{n=1}^{\infty} \frac{(-1)^n}{n} (\cos n\theta - \cos n(\pi - \theta)) \right] r^{-n-1} \quad (28)
\end{aligned}$$

Analyzing the stresses in the original Z plane (x, y) coordinates, we use

$$w(r, \theta) = w\left(e^{\frac{\pi}{a}x}, \frac{\pi}{a}y\right) = \phi(x, y) \quad (29)$$

$$\begin{aligned}
\phi(x, y) &= \frac{aT}{\mu\pi} \left[\frac{\pi}{2} - \frac{\pi}{a}y + \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos n \frac{\pi}{a}y - \cos n \pi \left(1 - \frac{y}{a}\right) \right] e^{\frac{n\pi x}{a}}, \quad e^{\frac{n\pi x}{a}} < 1 \\
&= \frac{aT}{\mu\pi} \left[\frac{1}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos n \frac{\pi}{a}y - \cos n \pi \left(1 - \frac{y}{a}\right) \right] e^{-\frac{n\pi x}{a}}, \quad e^{-\frac{n\pi x}{a}} > 1 \quad (30)
\end{aligned}$$

Hence

$$\frac{\partial \phi(x, y)}{\partial x} = \frac{nT}{\mu} \left[\frac{\pi}{2} - \frac{\pi}{a}y + \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos n \frac{\pi}{a}y - \cos n \pi \left(1 - \frac{y}{a}\right) \right] e^{\frac{n\pi x}{a}} \quad (31)$$

$$\begin{aligned}
\frac{\partial \phi\left(x, \frac{a}{2}\right)}{\partial x} &= \frac{nT}{\mu} \left[\frac{\pi}{2} - \frac{\pi}{a} \times \frac{a}{2} + \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos n \frac{\pi}{a} \times \frac{a}{2} - \cos n \pi \left(1 - \frac{1}{a} \times \frac{a}{2}\right) \right] e^{\frac{n\pi x}{a}} \\
&= 0
\end{aligned}$$

Therefore

$$\sigma_{xz}\left(x, \frac{a}{2}\right) = 0 \quad (32)$$

Also

$$\begin{aligned}
\frac{\partial \phi(x, y)}{\partial y} &= \frac{aT}{\mu\pi} \left[0 - \frac{\pi}{a} + \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \left(-\frac{n\pi}{a} \sin n \frac{\pi}{a} y - \frac{n\pi}{a} \sin n\pi \left(1 - \frac{y}{a} \right) \right) \right] e^{\frac{n\pi x}{a}} \\
&= \frac{aT}{\mu\pi} \left[-\frac{\pi}{a} - \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \left(\sin \frac{n\pi}{a} y + \sin n\pi \left(1 - \frac{y}{a} \right) \right) \right] e^{\frac{n\pi x}{a}} \\
&= -\frac{T}{\mu\pi} \left[\pi + \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \left(\sin \frac{n\pi}{a} y + \sin n\pi \left(1 - \frac{y}{a} \right) \right) \right] e^{\frac{n\pi x}{a}} \\
\frac{\partial \phi(x, y)}{\partial x} &= \frac{nT}{\mu} \left[\frac{\pi}{2} - \frac{\pi}{a} y + \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos n \frac{\pi}{a} y - \cos n\pi \left(1 - \frac{y}{a} \right) \right] e^{\frac{n\pi x}{a}} \\
&= -\frac{T}{\mu\pi} \left[\pi + \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \left(\sin \frac{n\pi}{2} + \sin \frac{n\pi}{2} \right) \right] e^{\frac{n\pi x}{a}}
\end{aligned}$$

Therefore

$$\sigma_{yz} \left(x, \frac{a}{2} \right) = \mu \frac{\partial w \left(x, \frac{a}{2} \right)}{\partial y} = -\frac{T}{\pi} \left[\pi + 2 \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin \frac{n\pi}{2} \right] e^{\frac{n\pi x}{a}} \quad (33)$$

For $e^{\frac{n\pi x}{a}} > 1$

$$\begin{aligned}
\phi(x, y) &= \frac{aT}{\mu\pi} \left[\frac{1}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \left(\cos \frac{n\pi}{a} y - \cos n \left(\pi - \frac{\pi}{a} y \right) \right) \right] e^{\frac{n\pi x}{a}} \\
\frac{\partial \phi(x, y)}{\partial x} &= \left(-\frac{n\pi}{a} \right) \frac{aT}{\mu\pi} \left[\frac{1}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \left(\cos \frac{n\pi}{a} y - \cos n \pi \left(1 - \frac{1}{a} y \right) \right) \right] e^{\frac{n\pi x}{a}} \\
\frac{\partial \phi \left(x, \frac{a}{2} \right)}{\partial x} &= \left(-\frac{n\pi}{a} \right) \frac{aT}{\mu\pi} \left[\frac{1}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \left(\cos \frac{n\pi}{a} \times \frac{a}{2} - \cos n \pi \left(1 - \frac{1}{a} \times \frac{a}{2} \right) \right) \right] e^{\frac{n\pi x}{a}} \\
&= -\frac{nT}{\mu} \left[\frac{1}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \left(\cos \frac{n\pi}{2} - \cos \frac{n\pi}{2} \right) \right] e^{\frac{n\pi x}{a}} = 0 \quad (34)
\end{aligned}$$

$$\therefore \sigma_{xz} \left(x, \frac{a}{2} \right) = 0 \quad (35)$$

Similarly

$$\begin{aligned}
\frac{\partial \phi(x, y)}{\partial y} &= \frac{aT}{\mu\pi} \left[\frac{1}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \left(-\frac{n\pi}{a} \sin \frac{n\pi}{a} y + n \sin n \left(\pi - \frac{\pi}{a} y \right) \left(-\frac{\pi}{a} \right) \right) \right] e^{\frac{n\pi x}{a}} \\
&= -\frac{aT}{\mu\pi} \left[\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \left(\sin \frac{n\pi}{a} y + \sin n \left(\pi - \frac{\pi}{a} y \right) \left(-\frac{\pi}{a} \right) \right) \right] e^{\frac{n\pi x}{a}}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial w\left(x, \frac{a}{2}\right)}{\partial y} &= -\frac{aT}{\mu\pi} \left[\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \left(\sin \frac{n\pi}{a} \times \frac{a}{2} + \sin n \left(\pi - \frac{\pi}{a} \times \frac{a}{2} \right) \left(-\frac{\pi}{a} \right) \right) \right] e^{-\frac{n\pi x}{a}} \\
&= -\frac{aT}{\mu\pi} \left[\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \left(\sin \frac{n\pi}{2} + \sin n \frac{\pi}{2} \right) \right] e^{-\frac{n\pi x}{a}} \\
&= -2 \frac{aT}{\mu\pi} \left[\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin \frac{n\pi}{2} \right] e^{-\frac{n\pi x}{a}}
\end{aligned} \tag{36}$$

$$\therefore \sigma_{yz}\left(x, \frac{a}{2}\right) = \mu \frac{\partial \phi\left(x, \frac{a}{2}\right)}{\partial y} = -\frac{2aT}{\pi} \left[\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin \frac{n\pi}{2} \right] e^{-\frac{n\pi x}{a}} \tag{37}$$

The displacement at $y = \frac{a}{2}$, $e^{-\frac{n\pi x}{a}} < 1$

$$\begin{aligned}
\phi(x, y) &= \frac{aT}{\mu\pi} \left[\frac{\pi}{2} - \frac{\pi}{a} y + \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \left(\cos n \frac{\pi}{a} y - \cos n \left(\pi - \frac{\pi}{a} y \right) \right) \right] e^{-\frac{n\pi x}{a}} \\
\phi\left(x, \frac{a}{2}\right) &= \frac{aT}{\mu\pi} \left[\frac{\pi}{2} - \frac{\pi}{a} \left(\frac{a}{2} \right) + \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \left(\cos \frac{n\pi}{a} \left(\frac{a}{2} \right) - \cos n \left(\pi - \frac{\pi}{a} \left(\frac{a}{2} \right) \right) \right) \right] e^{-\frac{n\pi x}{a}} \\
&= \frac{aT}{\mu\pi} \left[\frac{\pi}{2} - \frac{\pi}{2} + \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \left(\cos \frac{n\pi}{2} - \cos n \left(\pi - \frac{\pi}{2} \right) \right) \right] e^{-\frac{n\pi x}{a}} \\
&= 0
\end{aligned} \tag{37}$$

The displacement at $y = \frac{a}{2}$, $e^{-\frac{n\pi x}{a}} > 1$

$$\begin{aligned}
w\left(x, \frac{a}{2}\right) &= \frac{aT}{\mu\pi} \left[\frac{1}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \left(\cos \frac{\pi}{a} \left(\frac{a}{2} \right) n - \cos n \left(\pi - \frac{\pi}{a} \left(\frac{a}{2} \right) \right) \right) \right] e^{-\frac{n\pi x}{a}} \\
&= \frac{aT}{3\mu\pi} \left[\frac{1}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \left(\cos \frac{\pi}{2} n - \cos \frac{\pi}{2} n \right) \right] e^{-\frac{n\pi x}{a}} \\
&= 0
\end{aligned} \tag{38}$$

4.0 Conclusion

In this paper, the crack initiation point for an isotropic layer under out of plane stress is investigated and established to be at the origin of the layer. The novelty of constructing a holomorphic function that mapped the complex geometry of the z plane into a upper half plane as shown in Fig.2, enabled the use integral transform method to obtain an analytic solution of the displacement field.

The results show that since the shearing stress

$\sigma_{yz}\left(x, \frac{a}{2}\right)$ depends on the applied tractions T , it

follows that cracking will initiate at the origin of the layer if T becomes very large. This information furnishes the manufacturer as well as the user of the isotropic layer or appliance to get a crack breaker handy and at the right position of likely crack initiation.

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